# Biomedical Application of Mathematics

Masters course in Mathematics

Specialism: *Modelling and Simulation for Biomedical Applications*

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# Liang's model

Matlab 2015b

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Reference : Bonmassari's slides, Liang et al. [1], Lecture notes of Computational Haemodynamics [2] and Strocchi et al. [3]

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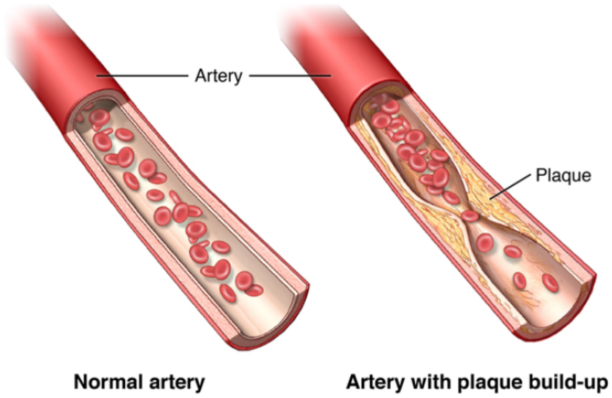
# Liang's model

We are interested in studying the haemodynamic effects of an arterial stenosis in a global model of the human circulation.



# Liang's model

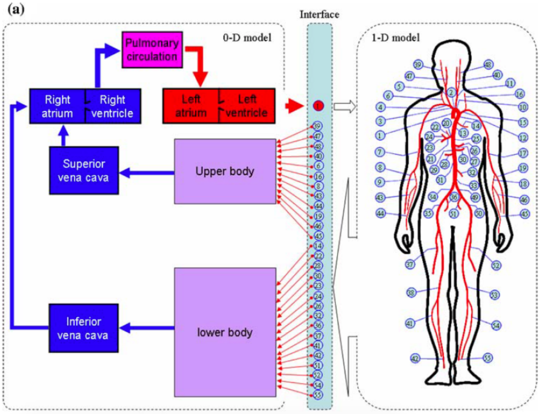



Figure : From Liang et al. [1]

The 1D model is composed by 55 arteries.

**Table 1** Physiological data of the arterial tree

| No. | Arterial segment | $L$ (cm) | $r_0$ (cm) | $r_1$ (cm) | $c_0$ (m $s^{-1}$) | $R_0$ (mmHg s $ml^{-1}$) | $R_1$ (mmHg s $ml^{-1}$) | $C_1$ (ml $mmHg^{-1}$) |
|---|---|---|---|---|---|---|---|---|
| 1 | Ascending aorta | 2.0 | 1.525 | 1.420 | 5.11 | – | – | – |
| 2 | Aortic arch I | 3.0 | 1.420 | 1.342 | 5.11 | – | – | – |
| 3 | Brachiocephalic | 3.5 | 0.650 | 0.620 | 5.91 | – | – | – |
| 4 | R.subclavian I | 3.5 | 0.425 | 0.407 | 5.29 | – | – | – |
| 5 | R.carotid | 17.7 | 0.400 | 0.370 | 5.92 | – | – | – |
| 6 | R.vertebral | 13.5 | 0.200 | 0.200 | 9.64 | 6.10 | 27.87 | 0.0126 |
| 7 | R.subclavian II | 39.8 | 0.407 | 0.230 | 5.38 | – | – | – |
| 8 | R.radius | 22.0 | 0.175 | 0.140 | 10.12 | 14.21 | 18.34 | 0.0143 |
| 9 | R.ulnar I | 6.7 | 0.215 | 0.215 | 8.78 | – | – | – |
| 10 | Aortic arch II | 4.0 | 1.342 | 1.246 | 5.11 | – | – | – |
| 11 | L.carotid | 20.8 | 0.400 | 0.370 | 5.92 | – | – | – |
| 12 | Thoracic aorta I | 5.5 | 1.246 | 1.124 | 5.11 | – | – | – |
| 13 | Thoracic aorta II | 10.5 | 1.124 | 0.924 | 5.11 | – | – | – |
| 14 | Intercoastals | 7.3 | 0.300 | 0.300 | 7.13 | 2.00 | 6.04 | 0.0542 |

Figure : Table n. 1 of [1]

We have modeled the healthy control (no stenosis), two stenoses: in the ascending aorta (no. 1) and in the thoracic aorta (n. 13).

# Exam

You are asked to prepare an oral presentation with the following features:

- Characterize the cardiovascular system using the indicators introduced by Dr. Bonmassari: PV loop, mean arterial pressure, CI, etc.
- Discuss the differences between the healthy control and patients with the stenosis.
- Prepare an oral presentation (15-20 minutes) with slides.
- Create a zip file containing the slides and the scripts you have used to study the data, call it **yoursurname-BAM-2015-16.zip** and submit to christian.contarino@unitn.it and eleuterio.toro@unitn.it one week before the exam.

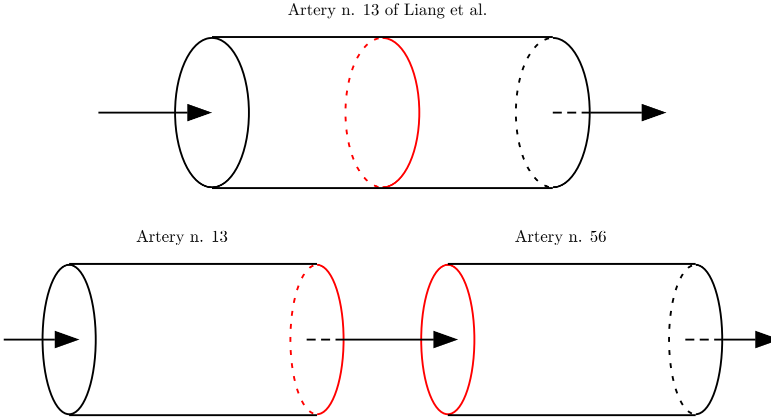
# Data

- You have 3 folders : HealthyControl, StenosisAscAo and StenosisThorAo
  - HealthyControl : healthy control
  - StenosisAscAo : Stenosis in the ascending aorta (Artery No. 1).
  - StenosisThorAo : Stenosis in the thoracic aorta (Artery No. 13)
- Each folder contains dat-files : NameA.dat, NameP.dat, NameQ.dat and NameHeart.dat

## Important

- Assume 77-kg person of 1.75 $m$ in height.
- A cardiac cycle lasts 1 second.
- You have 55+1 arteries

# Stenosis in the thoracic aorta



# Summary of the data : 1D model

- **NameP.dat**, **NameA.dat**, **NameQ.dat**
  - First column is time *s*
  - From column 2 to column 56, you find pressures, cross-sectional areas and flows in arteries from 1 to 55 of table 1 of Liang et al. [1]
  - Column 57 represents the second half of the vessel with the stenosis
    - For HealthyControl do not consider the 56th artery
    - Note that for the stenosis in the ascending aorta, artery no. 1 of [1], is composed of two vessels, whose column numbers are 2 (first proximal half) and 57 (second proximal half)
    - Note that for the stenosis in the thoracic aorta, artery no. 13 of [1] is composed of two vessels, whose column numbers are 14 (first proximal half) and 57 (second proximal half)
  - Pressure is measured in $mmHg$, cross-sectional area in $m^2$ and flow in $m^3/s$
  - The data is given by considering sampling area and flow at the center of each artery

# Summary of the data : 0D model

- **NameHeart.dat**
  - First column is the time $s$
  - Pressure is measured in $mmHg$, volume in $mL$ and flow in $mL/s$

| Column | | |
|---|---|---|
| 10 | $V_{RA}$ | Volume right atrium |
| 11 | $Q_{TV}$ | Flow through the tricuspid valve |
| 12 | $V_{RV}$ | Volume right ventricle |
| 13 | $Q_{PV}$ | Flow through the pulmonary valve |
| 20 | $V_{LA}$ | Volume left atrium |
| 21 | $Q_{MV}$ | Flow through the mitral valve |
| 22 | $V_{LV}$ | Volume left ventricle |
| 23 | $Q_{AV}$ | Flow through the aortic valve |
| 32 | $P_{LA}$ | Pressure left atrium |
| 33 | $P_{LV}$ | Pressure left ventricle |
| 34 | $P_{RA}$ | Pressure right atrium |
| 35 | $P_{RV}$ | Pressure right ventricle |

If you think you need other data (such as volume in the lower capillaries), write me.

# Useful commands in matlab

- Load the Healthy control's pressure

```
P=load('HealthyControl/SaveData/HealthyControlP.dat') %Load the .dat file
```

  - P(:,1) is the time. Time goes from 9 to 10 because we have simulated 10 cardiac cycles.
  - P(:,2) is the pressure of the first artery of the Liang's model, namely the ascending aorta.

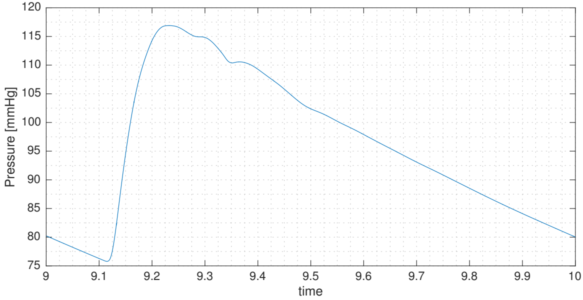
# Useful commands in matlab

- Plot the pressure of the 13th artery

```
plot(P(:,1),P(:,14))  %P(:,1) is the time and P(:,14) is the pressure of the 13th artery
```

- Mean value of the 13th artery

```
mean(P(:,14))
ans =
  96.0276
```



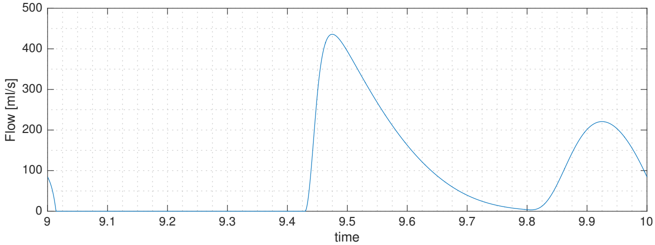
# Useful commands in matlab

- Load the Heart information of the Healthy control

```
H=load('HealthyControl/SaveData/HealthyControlHeart.dat')
%Load the .dat file
```

- Plot the flow through the mitral valve

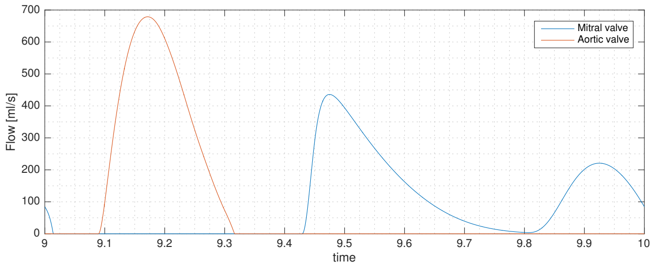
```
plot(H(:,1),H(:,21))  %H(:,1) is the time and H(:,21) is the flow through the mitral valve
```



# Useful commands in matlab

- 2 plots in the same figure with a grid and a legend.

```
plot(H(:,1),H(:,21))
grid minor
xlabel('time')
ylabel('Flow [ml/s]')
hold on
plot(H(:,1),H(:,23))
legend('Mitral valve','Aortic valve')
```



# Indicators

Here is an example of possible indicators you can use to characterize the cardiovascular system

- Pressure pulse
- Stroke volume and stroke volume index
- Cardiac output and cardiac index
- Ending diastolic and systolic volume
- ABI
- Plot the pressure of the ascending aorta, left atrium and left ventricle
- Refer to the plots of [1] and [4]

Use plots and histograms to show the differences.

# References


F. Liang, S. Takagi, R. Himeno, H. Liu, Multi-scale modeling of the human cardiovascular system with applications to aortic valvular and arterial stenoses, Med Biol Eng Comput 47 (7) (2009) 743–755.
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